

2.1 Derivative Basics: Form G Key

Tutor copy. Every answer was checked by symbolic computation (SymPy, `verify_FJ.py`). All problems are original; provenance is in `PROVENANCE_GJ.md`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Answer-only items** (Q1, Q9): all or nothing, except where a row below says otherwise.
- **Value vs. derivative.** Giving a derivative where a function value was asked (or the reverse), or a number from the wrong column of a table, earns 0 for that part unless a row below says otherwise.
- **Show-work items** (Parts II–IV other than Q9, and Part VI): a bare correct answer earns at most half credit.
- **Definition items** (Q6, Q7): the problem states the required method. A correct answer by differentiation rules earns at most 2 of the 8 points.
- **Method is not graded for efficiency.** A correct answer reached by long but valid work gets full credit; the time it costs is the penalty.

Form G

Item	Answer	Rubric / trap
1(a)	3	<i>Trap:</i> the tangent drawn there invites the slope, 2; earns 0.
1(b)	-2	<i>Trap:</i> giving the height $f(-3) = 3$; earns 0.
1(c)	0	<i>Trap:</i> giving the height 1 of the low point; earns 0.
1(d)	DNE	Corner at $(1, 3)$: one-sided slopes 2 and -2. <i>Trap:</i> 0, because $(1, 3)$ is a high point.
1(e)	$(-4, -1)$ and $(1, 2)$	3 pts, all or nothing; union notation is fine. Endpoints may not be included. <i>Trap:</i> $(-4, 2)$, which includes the rising stretch $(-1, 1)$ and the corner.
1(f)	$f'(-3) < 0 < f'(3)$	$f'(-3) = -2$, $f'(3) = 2$. 3 pts, all or nothing.
2	$y - 2 = -9(x - 1)$, i.e. $y = -9x + 11$	1 pt $r(1) = 2$; 2 pts $r'(x)$; 1 pt $r'(1) = -9$; 1 pt equation in any correct form. <i>Trap:</i> $r'(x) = 3x^2 - 12x$, so $r'(1) = 3 - 12 = -9$; the $-6x^2$ term differentiates to $-12x$, not -12 . Using $r(1) = 2$ as the slope earns 2 of 5.
3	$\frac{3}{4}$	2 pts $q(3) = 4$; 3 pts slope $\frac{4-1}{3-(-1)} = \frac{3}{4}$. <i>Trap:</i> q needs the chain rule, which has not been taught, so the given point is the only route. Students who try to differentiate q stall. Sign slips with $3 - (-1)$ earn 2.
4(a)	3	3 pts. Simplifies to $y = 3x - 3$. <i>Trap:</i> -2, the coefficient in the familiar slope position; earns 0.
4(b)	-12	2 pts. Substitute $x = -3$. <i>Trap:</i> -1, read from the left side; earns 0.
5	5	<i>Trap:</i> s' is printed and invites $s'(1) = 8$; earns 0.
6	$f'(x) = -\frac{12}{(3x+2)^2}$	2 pts $f(x+h) = \frac{4}{3(x+h)+2} = \frac{4}{3x+3h+2}$, substituted correctly and in parentheses; 2 pts common denominator, numerator $4(3x+2) - 4(3x+3h+2) = -12h$; 2 pts cancel h to get $\frac{-12}{(3x+3h+2)(3x+2)}$; 2 pts let $h \rightarrow 0$. <i>Trap:</i> writing $f(x) + h$ for $f(x+h)$, or losing a sign across the subtraction. Rules only: at most 2.

Item	Answer	Rubric / trap
7	16	1 pt $g(-2) = -16$; 1 pt setup; 3 pts factoring out $x + 2$ to get $x^2 - 2x + 8$, by any valid method. Synthetic or long division by $x + 2$ works (on 1, 0, 4, 16, with the zero placeholder). So does grouping with the sum of cubes: $x^3 + 4x + 16 = (x^3 + 8) + 4(x + 2) = (x + 2)(x^2 - 2x + 4) + 4(x + 2) = (x + 2)(x^2 - 2x + 8)$. No particular method is required. 3 pts value. <i>Trap</i> : $g(-2) = -8 - 8 = -16$, so subtracting it gives $+16$; the denominator is $x - (-2) = x + 2$. A student who divides and drops the $0x^2$ placeholder gets a wrong quotient. Rules only: at most 2.
8(a)	$\frac{\sqrt{3}}{2}$	$\frac{1}{2} = \sin \frac{\pi}{6}$, so this is the derivative of $\sin$ at $\frac{\pi}{6}$: $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$. <i>Trap</i> : “0/0, DNE” or $\frac{1}{2}$; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
8(b)	12	$-8 = (-2)^3$, so this is the derivative of x^3 at -2 : $3(-2)^2 = 12$. <i>Trap</i> : -12 , from $(-2)^2 = -4$; earns 0. 1 pt for recognizing which derivative the limit is (or a correct algebraic setup), 2 pts for the value.
9	(a) 0 (b) 0	3 each. <i>Traps</i> : (a) $4 \cos(\pi/3) = 2$; (b) $f'(-1) = -9$. In (b), $m(x)$ is the same number for every x , so $m'(x) = 0$; the problem says “for every x ”.
10(a)	$-32x^{-5} + 3 \sin x$	1 pt rewriting $8x^{-4}$; 2 pts derivative. <i>Trap</i> : the exponent $-4 - 1 = -5$; the derivative of $-3 \cos x$ is $+3 \sin x$.
10(b)	$4x$	1 pt rewriting $2x^2 - 3$ (for $x \neq 0$); 2 pts derivative. <i>Trap</i> : $\frac{9x^2}{3x^2} = 3$, a constant; differentiating top and bottom separately earns 0.
10(c)	$6x^{1/2} - 3x^{-1/2}$	1 pt rewriting $4x^{3/2} - 6x^{1/2}$; 2 pts derivative. <i>Trap</i> : $x\sqrt{x} = x^{3/2}$; $6 \cdot \frac{1}{2} = 3$.
10(d)	$4x - 5 + 5 \cos x$	1 pt rewriting $2x^2 - 5x - 3 + 5 \sin x$; 2 pts derivative. <i>Trap</i> : the product expands to $2x^2 - 5x - 3$; multiplying the derivatives $2 \cdot 1$ is not a rule.
11	$\frac{4\pi}{3} - \frac{3\sqrt{3}}{2}$	3 pts $f'(x) = -3 \sin x + 2x$; 2 pts value. <i>Trap</i> : $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$; the derivative of $3 \cos x$ is $-3 \sin x$. Using $+\frac{3\sqrt{3}}{2}$ or $\cos \frac{2\pi}{3}$ earns 3.
12	$x = \frac{7\pi}{6}$	2 pts $f'(x) = 1 + 2 \sin x$; 2 pts $\sin x = -\frac{1}{2}$; 2 pts the solutions in the interval. <i>Trap</i> : $\frac{11\pi}{6}$ also solves $\sin x = -\frac{1}{2}$ but lies outside $(\pi, \frac{3\pi}{2}]$; listing it costs 2.
13	II	6 pts, all or nothing. In II the dashed curve is $f(x) = x^4 - 2x^2$ and the solid curve is $f'(x) = 4x^3 - 4x$: it is zero exactly where the dashed curve turns ($x = -1, 0, 1$), positive where that curve rises, and negative where it falls. Wrong choices: I: right zeros, wrong sign: it is positive where the W-shaped curve falls (for example just left of $x = 1$); III: shifted: the other curve is zero at $x = 0, 1, 2$, not at the turning points $x = -1, 0, 1$; IV: the other curve is zero where the W-shaped curve crosses the x -axis ($x = 0, \pm\sqrt{2}$), not where it turns. <i>Trap</i> : assuming the dashed curve is always the derivative; the styles swap between graphs.
14(a)	-10	$h'(x) = 3f'(x) - 2x$, so $h'(2) = 3(-2) - 2(2)$. Read the <i>height</i> of the f' graph. Heights at the marked points: $f'(-3) = 2$, $f'(-1) = 0$, $f'(1) = -3$, $f'(2) = -2$, $f'(3) = 0$. 2 pts reading $f'(2)$, 2 pts value. <i>Trap</i> : using the slope of the f' graph at 2 ($\frac{3}{2}$) gives $\frac{1}{2}$; earns 0.
14(b)	$x = -1$ and $x = 3$	Horizontal tangents of f are where $f' = 0$: the x -intercepts of the f' graph. All needed; all or nothing. <i>Trap</i> : $x = 1$, the lowest point of the f' graph (where f' , not f , is flat); earns 0.
14(c)	$x = 1$	The slope of f 's tangent line at x is $f'(x)$, the height of this graph; it is least at the lowest point of the graph, $x = 1$. All or nothing. <i>Traps</i> : the x -intercepts of the f' graph (where f is flat); the place where the f' graph is steepest.

Points: $14 + 5 + 5 + 5 + 2 + 8 + 8 + 6 + 6 + 12 + 5 + 6 + 6 + 12 = 100$.